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CORRELATING EQUATIONS FOR LAMINAR AND TURBULENT FREE CONVECTION FROM A VERTICAL PLATE

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Abstract—A simple expression is developed for the space-mean *Nu* (or *Sh*) for all *Ra* and *Pr* (or *Sc*) in terms of the model of Churchill and Usagi. The development utilizes experimental values for *Ra* approaching zero and infinity, and the theoretical solutions obtained from laminar boundary-layer theory. The expression is applicable to uniform heating as well as to uniform wall temperature and for mass transfer and simultaneous heat and mass transfer. The correlation provides a basis for estimating transfer rates for non-Newtonian fluids and for inclined plates. Even simpler expressions are developed for restricted ranges of conditions. The general and restricted expressions are compared with representative experimental data. The structure of the correlating equation shows why the common power-law-type equations cannot be successful over an extended range of *Ra* and *Pr*.

NOMENCLATURE

a, arbitrary exponent;
A, dimensionless coefficient;
b, arbitrary exponent;
c, dimensionless coefficient;
$\mathscr{D}$, diffusivity [m²/s];
$f\{Pr\}$, dimensionless function of Pr in equation (2);
$F\{m\}$, dimensionless function of power-law coefficient in equation (16);
g, acceleration due to gravity [m/s²];
h, local heat-transfer coefficient [J/m².s.°K];
$\bar{h}$, mean heat-transfer coefficient over 0–z [J/m².s.°K];
k, thermal conductivity [J/m.s.°K];
k', local mass-transfer coefficient [s⁻¹];
$\bar{k}'$, mean mass-transfer coefficient over 0–z [s⁻¹];
K, coefficient defined by equation (15) [kg/m-s^{2-m}];
m, exponent defined by equation (15);
n, exponent in equation (1);
Nu, hz/k, local Nusselt number at z;
$\overline{Nu}$, $\bar{h}z/k$, mean Nusselt number over 0–z;
Pr, ν/α, Prandtl number;
q, heat flux density [J/m²-s];
Ra, $g\beta(T_s-T_b)z^3/\nu\alpha$, Rayleigh number;
Ra', $g\gamma(\omega_s-\omega_b)z^3/\nu\mathscr{D}$, Rayleigh number for mass transfer;
Ra^*, $g\beta qz^4/k\nu\alpha$, modified Rayleigh number based on heat flux density;
Sc, $\nu/\mathscr{D}$, Schmidt number;
Sh, $k'z/\mathscr{D}$, local Sherwood number;
$\overline{Sh}$, $\bar{k}'z/\mathscr{D}$, mean Sherwood number over 0–z;
T, temperature [°K];
x, independent variable [m];
y, dependent variable [m];
z, distance up plate [m].

Greek symbols

α, thermal diffusivity [m²/s];
β, thermal coefficient of expansion [°K⁻¹];
γ, dimensionless coefficient for expansion due to change in composition;
ω, mass fraction;
ν, kinematic viscosity [m²/s];
$\varphi\{Pr\}$, dimensionless function of Pr in equation (8);
θ, angle of inclination of the plate from the vertical;
τ, shear stress [kg/m-s²].

Subscripts

b, bulk;
s, surface;
0, limiting behavior for small z;
∞, limiting behavior for large z.

INTRODUCTION

A VARIETY of theoretical expressions, graphical correlations and empirical equations have been developed to represent the coefficients for heat and mass transfer by free convection from vertical plates. However, the discrepancies between the expressions proposed for correlation and the various sets of experimental data have still not been completely resolved or explained. The experimental anomalies are apparently due in part to physical property variations and undefined differences in the environment. The theoretical results are mostly limited to the intermediate range of Rayleigh number for which the postulates of laminar boundary-layer theory are applicable; a completely satisfactory theory has not been developed for either the diffusive regime (low Rayleigh numbers) or the turbulent regime (high Rayleigh numbers). The primary shortcoming of the empirical correlations is their failure to take into proper account the *varying* dependence on the Rayleigh and Prandtl (or Schmidt) numbers.

This paper presents simple but very general correlations for the space–mean value of the transfer rate for free convection. The correlations are developed wholly in terms of the model of Churchill and Usagi [1]:

$$y^n\{z\} = y_0^n\{z\} + y_\infty^n\{z\} \tag{1}$$

and thus require appropriate expressions for the limiting behavior for both large and small values of the independent variable z.

Ede [2] provides a thorough review of the literature for heat transfer through 1964. In the interest of brevity, correlations, theoretical solutions and experimental data since that date will not be reviewed or analyzed except insofar as they are directly relevant to the derivations herein. The correlation is first developed in terms of heat transfer from an isothermal plate. Uniform heating, mass transfer, simultaneous heat and mass transfer, non-Newtonian fluids and inclined plates are subsequently considered.

LAMINAR REGIME

Boundary-layer theory has been utilized to derive relationships of the form:

$$Nu = Ra^{1/4} f\{Pr\} \tag{2}$$

where $f\{Pr\}$ represents a tabulation of values such as those summarized by Ede [2] for a number of values of Pr. Churchill and Usagi [1] derived an empirical expression in the form of equation (1) to provide a continuous approximation for these tabulated values of $f\{Pr\}$. This expression can be rewritten as follows in terms of $\overline{Nu}$:

$$\overline{Nu} = 0{\cdot}670 Ra^{1/4}/[1+(0{\cdot}492/Pr)^{9/16}]^{4/9}. \tag{3}$$

Equation (3) represents the various computed values within 1 per cent from $Pr = 0$ to $Pr = \infty$ and is in general agreement for $10^5 < Ra < 10^9$ with the widely scattered experimental values compiled by Ede [2].

Equation (2) and hence equation (3) would be expected to become invalid for $Ra > 10^9$ owing to the onset of turbulence and as $Ra \to 0$ owing to thickening of the boundary layer relative to the distance from the starting edge of the plate. A generally accepted solution has not been derived for this latter regime. For pure conduction ($Ra = 0$) from an infinite strip $\overline{Nu} = 0$, but for a plate of finite dimensions $\overline{Nu}$ has a finite value. The experimental data of Saunders [5] indicate a limiting value of approximately 0·68, probably due to edge effects.

Utilizing 0·68 for $y_0\{z\}$ and the right side of equation (3) for $y_\infty\{z\}$ in equation (1) yields the following test expression for the entire laminar regime:

$$\overline{Nu}^n = 0{\cdot}68^n + \left(\frac{0{\cdot}670 Ra^{1/4}}{[1+(0{\cdot}492/Pr)^{9/16}]^{4/9}}\right)^n. \tag{4}$$

A test plot of representative experimental data [2–13] in the form proposed by Churchill and Usagi [1] indicates that $n = 1$ is a reasonable choice, yielding

$$\overline{Nu} = 0{\cdot}68 + \frac{0{\cdot}670 Ra^{1/4}}{[1+(0{\cdot}492/Pr)^{9/16}]^{4/9}}. \tag{5}$$

Equation (5) is seen in Fig. 1 to provide a good representation for all $Ra < 10^9$ while equation (3) is seen to be increasingly in error for $Ra < 10^5$.

LAMINAR PLUS TURBULENT REGIME

An asymptotic solution is not available for $Ra \to \infty$, but Churchill [14] has asserted on the basis of dimensional analysis that

$$Nu \to A Ra^{1/3} \varphi\{Pr\} \tag{6}$$

where A is an empirical constant and $\varphi\{Pr\}$ is a function which approaches unity for $Pr \to \infty$ and is proportional to $Pr^{1/3}$ for $Pr \to 0$. Equations (5) and (6) could be combined in the form of equation (1) to obtain a test expression for all Ra and Pr. However the limiting value of 0·68 proves to combine with equation (6) to produce a simpler and equally successful correlation. The resulting test expression is

$$\overline{Nu}^n = 0{\cdot}68^n + [A Ra^{1/3} \varphi\{Pr\}]^n. \tag{7}$$

Equation (7) provides a dependence of Nu on Ra for any positive n which increases continuously from the zeroth power to the 1/3-power as Ra increases. If equation (7) is to provide the same interrelationship between Ra and Pr in the laminar boundary-layer regime as equation (5) it is necessary that:

$$\varphi\{Pr\} = ([1+(0{\cdot}492/Pr)^{9/16}]^{-4/9})^{4/3} = [1+(0{\cdot}492/Pr)^{9/16}]^{-16/27} \tag{8}$$

The expression resulting from insertion of equation (8) in (7) also conforms to the asserted dependence for $Pr \to 0$ and ∞ as $Ra \to \infty$.

Bosworth [15] proposed an equation of the form of equation (7) with $\varphi\{Pr\} = 1{\cdot}0$ and $n = 1/2$ for $\overline{Nu}$ for free convection from horizontal cylinders in air. Trial plots indicate that $n = 1/2$ is a reasonable choice for the vertical plate as well. The straight line with a slope of 1/6 drawn in Fig. 2 through the same representative data as in Fig. 1 yields a value of $A = 0{\cdot}150$ and hence the final correlation:

$$\overline{Nu}^{1/2} = 0{\cdot}825 + \frac{0{\cdot}387 Ra^{1/6}}{[1+(0{\cdot}492/Pr)^{9/16}]^{8/27}}. \tag{9}$$

This value of A is in reasonable accord with the value of $A\varphi\{Pr\} = 0{\cdot}10$, hence $A = 0{\cdot}12$, derived by Bayley [16] for air and also with the value of 0·13 proposed by Kutateladze [17] for a correlation in the form of equation (6) for turbulent free convection from vertical plates, cylinders and spheres to a number of fluids.

Equations (3) and (5) are plotted also in Fig. 2 for comparison and to indicate their limits of applicability. The undoubted superiority of equation (9) for $Ra > 19^9$ is somewhat obscured by the lack of data for truly high Ra, the scatter of the available data and the very condensed scale of the ordinate.

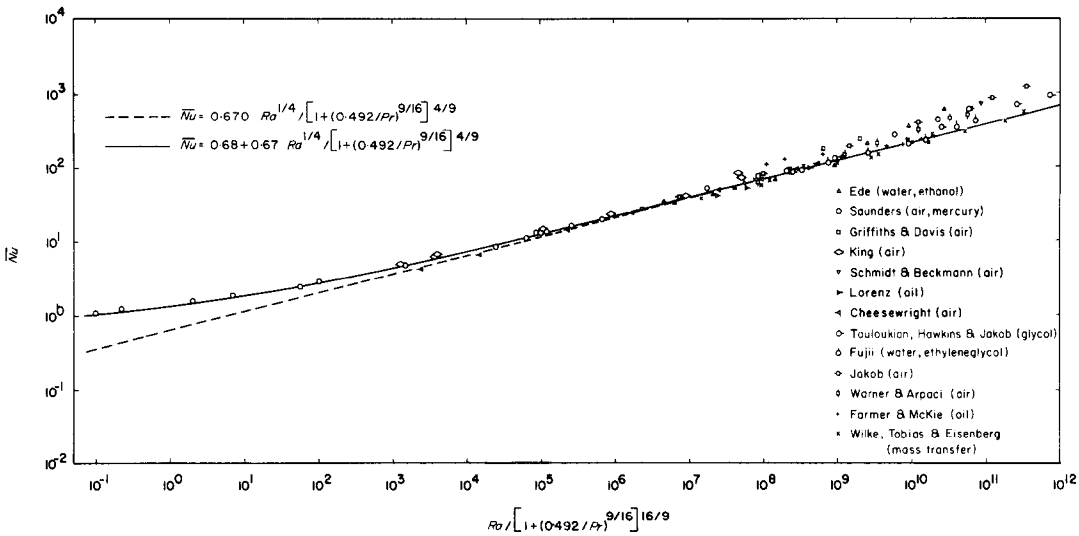

FIG. 1. Correlating equations for the laminar regime of isothermal, vertical plates.

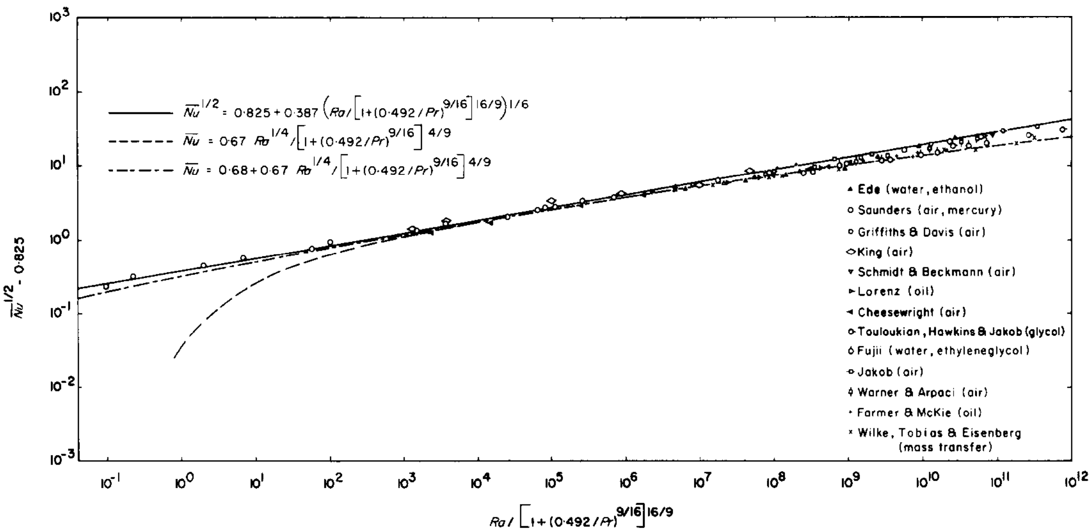

FIG. 2. Comparison of correlating equations with experimental data for isothermal, vertical plates.

INTERPRETATION

Computed values of $f\{Pr\}$ and $\varphi\{Pr\}$ for representative fluids are given in Table 1. The significant deviations for air and water from the limiting dependence for $Pr \to \infty$ indicate why the customary empirical equations of the form of equation (2) with $f\{Pr\} = 1{\cdot}0$ and equation (6) with $\varphi\{Pr\} = 1{\cdot}0$ have not proven satisfactory for a variety of fluids with a wide range of Pr. Table 1 also indicates that somewhat lesser but still significant discrepancies are to be expected with the simplified correlations for liquid metals based on the limiting form for $Pr \to 0$. A further variation in the dependence on Pr and Ra arises from the additive constant in equations (5) and (9). Thus empirical correlations of the form:

$$Nu = CRa^a Pr^b \tag{10}$$

cannot be expected to be successful over an extended range of Ra or Pr. Instead, the deviations from the correlations in the literature must be due in part to the choice of this form rather than wholly to experimental error. Such correlations appear to have outlived their usefulness.

Equation (9) provides a smooth transition from the laminar to the turbulent regime whereas the actual transition is known to be essentially discrete. The representation provided by equation (9) for this region is thus an oversimplification of reality and is numerically successful only because the effect of the transition is dampened by the integration which leads from the local to the mean Nusselt number. A correlation for the local Nusselt number extending through the transition from laminar to turbulent motion would need to be more complicated in structure than equation (9).

Table 1. Correction factor for various fluids from asymptotic behavior

Pr	Fluid	$f\{Pr\}$	$\varphi\{Pr\}$	$(0{\cdot}492/Pr)^{1/4}f\{Pr\}$	$(0{\cdot}492/Pr)^{1/3}\varphi\{Pr\}$
∞		1·000	1·000		
100	oil	0·978	0·971	0·259	0·165
7·0	water	0·914	0·887	0·471	0·366
0·70	air	0·766	0·701	0·702	0·623
0·024	mercury, 50°F	0·436	0·331	0·928	0·905
0·004	sodium, 1200°F	0·292	0·194	0·912	0·962
0				1·000	1·000

For large temperature differences such that the physical properties vary significantly, Ede [2] recommends that the physical properties be evaluated at the mean of the surface and the bulk temperature. Wylie [18] provides more detailed theoretical guidance for the laminar boundary-layer regime.

UNIFORM HEAT FLUX

The definition of the mean Nusselt number for uniform heating is somewhat arbitrary. However, Sparrow and Gregg [19] have shown that for a laminar boundary layer the use of the temperature difference at the midpoint of the plate yields values in better agreement with those for uniform wall temperature than the use of either the integrated mean temperature difference or the integrated mean heat-transfer coefficient. With this definition the following expression can be derived from the empirical representation of Churchill and Ozoe [20] for the local heat-transfer coefficient for uniform heating in a laminar boundary layer.

$$\overline{Nu} = 0{\cdot}670Ra^{1/4}/[1+(0{\cdot}437/Pr)^{9/16}]^{4/9}. \quad (11)$$

It may be noted that for $Pr \to \infty$ the coefficient of the Rayleigh number is indeed the same as that of equation (3) and that these expressions differ only by $((0{\cdot}492/0{\cdot}437)^{1/4}-1)100 = 3$ per cent even for $Pr \to 0$. [Equation (11) can be converted to one for the integrated mean temperature difference by multiplying the coefficient 0·670 by $(6/5)^{5/4}/2^{1/4}$ giving 0·708 and to the one for the integrated mean heat-transfer coefficient by multiplying by $(5/4)^{5/4}/2^{1/4}$ giving 0·745.]

Neither experimental data nor theoretical results appear to provide a limiting value of $\overline{Nu}$ for $Ra \to 0$. Hence the same value as for uniform wall temperature will arbitrarily be used. The exponent in equation (1) has generally been found to be the same for similar processes as illustrated by comparison of equations (3) and (11). Hence in the absence of experimental data the following expression is proposed for the entire laminar regime with uniform heating:

$$Nu = 0{\cdot}68 + \frac{0{\cdot}670Ra^{1/4}}{[1+(0{\cdot}437/Pr)^{9/16}]^{4/9}}. \quad (12)$$

An equation of the form of equation (6) would be expected to hold for uniform heating as well as uniform wall temperature. Combining equation (6) with $\overline{Nu}_0 = 0{\cdot}68$, forcing the same relationship between Ra and Pr as in equation (11) and assuming that 1/2 is again a satisfactory choice for n results in:

$$\overline{Nu}^{1/2} = 0{\cdot}825 + \frac{A^{1/2}Ra^{1/6}}{[1+(0{\cdot}437/Pr)^{9/16}]^{8/27}}. \quad (13)$$

A plot of a random selection from the limited sets of experimental data for uniform heating [21–24], in Fig. 3 in the form suggested by equation (13) again yields a value of $A = 0{\cdot}15$, producing the following correlation for uniform heating for all Ra and Pr:

$$\overline{Nu}^{1/2} = 0{\cdot}825 + \frac{0{\cdot}387Ra^{1/6}}{[1+(0{\cdot}437/Pr)^{9/16}]^{8/27}}. \quad (14)$$

Churchill [14] has asserted that Nu for fully developed turbulent motion ($Ra \to \infty$) should be the same for uniform heating as for wall temperature if a value independent of z, corresponding to a proportionality of Nu to $Ra^{1/3}$ is attained. This assertion is tested by plotting equation (9) for $Pr = 0{\cdot}70$ in Fig. 3. Good agreement with the data may be noted as would be expected since equations (9) and (14) differ only slightly in one coefficient.

Free convection with uniform heating is often correlated in terms of Ra^* in order to avoid explicit inclusion of the surface temperature. Equations (11), (12) and (14) can be rewritten in terms of Ra^* simply by replacing $\overline{T_s - T_b}$ with $q/\bar{h}$, hence Ra with $Ra^*/\overline{Nu}$. However, this re-expression disguises the important result that the dependence of $\overline{Nu}$ on Ra is essentially the same as for uniform wall temperature.

INCLINED SURFACES

Vliet [25] has reviewed prior results for inclined surfaces and presented additional results for uniform heating. He concludes that for the laminar regime the solutions and correlations for a vertical plate may be used for a plate inclined up to at least 60° from the vertical if the component of gravity parallel to the surface is used in the Rayleigh number. However, the Rayleigh number for transition from laminar to turbulent motion is decreased drastically as the angle of inclination from the vertical is increased and his local results for the turbulent regime were better correlated in terms of $\boldsymbol{g}$ than in terms of $\boldsymbol{g}\sin\varphi$.

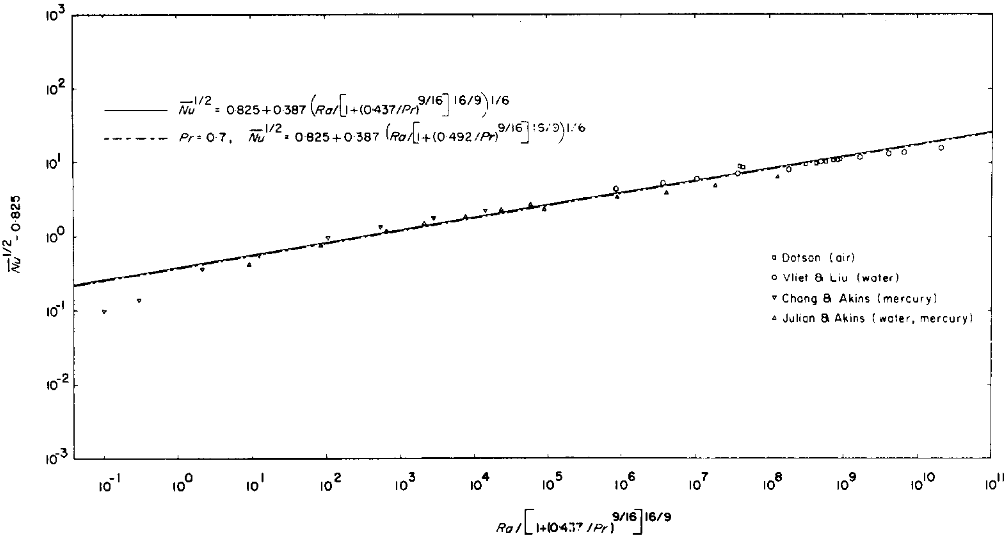

FIG. 3. Comparison of correlating equations with experimental data for uniformly heated, vertical plates.

MASS TRANSFER

Equations (5) and (9) with $\overline{Sh}$ substituted for $\overline{Nu}$, Sc for Pr, and Ra' for Ra are expected to hold for mass transfer as long as the net rate of mass transfer is not so high as to affect the velocity field significantly. Representative mass-transfer data [26] are included in Fig. 2 and reasonable agreement with equations (5) and (9) is apparent.

SIMULTANEOUS HEAT AND MASS TRANSFER

On the basis of the results of Saville and Churchill [27] and Lightfoot [28] for mass transfer due to a temperature gradient only $(\alpha(\omega_s - \omega_b)/\beta(T_s - T_b) \to 0$ and $Pr/Sc \to 0)$ $\overline{Sh}$ can be substituted for $\overline{Nu}$ and $Ra(Sc/Pr)^{4/3}$ for Ra in equation (9).

Also, on the basis of the results of Saville and Churchill [27] for simultaneous heat and mass transfer, $\overline{Nu}$ and Sh can be calculated from equation (9) for the special case of $Sc = Pr$ merely by substituting $Ra + Ra'$ for Ra. For $Sc \neq Pr$, the asymptotic solutions are not explicit and simple substitution in equation (9) is not possible [29].

NON-NEWTONIAN FLUIDS

For a power-law fluid such that:

$$\tau = -K\left|\frac{du}{dy}\right|^{m-1}\frac{du}{dy}. \tag{15}$$

Acrivos [30] has derived for $Pr \to \infty$ the following generalized form of equation (2):

$$Nu = F\{m\}\left(\frac{\rho g\beta(T_s - T_b)z^{2m+1}}{K\alpha^m\rho}\right)^{1/(3m+1)} \tag{16}$$

where $F\{m\}$ is a weak function of m and $F\{1.0\} = 0.670$. Equation (16) has been confirmed as a good representation for a number of fluids with $0.6 \leqslant m \leqslant 1.0$ by Agarwal *et al.* [30] for uniform wall temperature, and the analogue of equation (16) for uniform heating with $0.4 < m < 1.0$ by Chen and Wollersheim [32]. It follows that equation (5) with $f\{Pr\} = 1$ and equation (9) with $\varphi\{Pr\} = 1$ should be applicable for such fluids if $(\rho\beta(T_s - T_b)z^{2m+1}/K\alpha^m)^{4/3m+1}$ is susbsituted for Ra.

Fujii *et al.* [33] have obtained numerical solutions for a Sutterby fluid at finite Pr, and experimental results for aqueous solutions of polyethylene oxide. Their results indicate that Acrivos' solution may be a reasonable approximation for real fluids if the coefficients K and m are evaluated at the shear stress at the midheight of the heated plate.

CONCLUSIONS

1. Equation (9) based on the model of Churchill and Usagi provides a good representation for the mean heat transfer for free convection from an isothermal vertical plate over a complete range of Ra and Pr from 0 to ∞ even though it fails to indicate a discrete transition from laminar to turbulent flow.

2. Equation (14) provides an equivalent representation for heat transfer by free convection from a uniformly heated vertical plate. However, equation (9) is also an adequate representation for this boundary condition.

3. Equation (9) is applicable to mass transfer with $\overline{Sh}$, Ra' and Sc substituted for $\overline{Nu}$, Ra and Pr and can be applied for simultaneous heat and mass transfer for the special case of $Pr = Sc$ if $Ra + Ra'$ is substituted for Ra. Other such extensions are also possible.

4. More accurate representations for the laminar regime are provided by equations (5) and (12) and these simpler expressions should be used rather than equations (9) and (14) for $Ra < 10^9$. The expressions for the laminar regime are also applicable to mass transfer and simultaneous heat and mass transfer with the indicated substitutions.

5. Equations (5) and (12) are proposed as tentative representations for laminar convection from plates inclined up to at least 60° from the vertical if $g \sin\varphi$ is substituted for g. Based on the results of Vliet [25], equations (9) and (14) may be applicable for the turbulent regime without this modification. Fortunately

these equations are quite insensitive to the point of transition from laminar to turbulent motion.

6. Equations (9) with $Pr \to \infty$ is applicable to non-Newtonian fluids whose behavior can be represented by a power-law if $(\rho g \beta (T_s - T_b) z^{2m+1}/K\alpha^m)^{4/3m+1}$ is used for Ra.

7. The principal uncertainty in the correlations proposed herein arises from the uncertainty in the limiting solutions and experimental data for $Ra \to 0$ and ∞.

8. General correlations of the simple power-law type such as equation (10) are seen to be fundamentally unsound for any extended range of the variables and their use is no longer justified.

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LOIS DE CORRELATION EN CONVECTION NATURELLE LAMINAIRE ET TURBULENTE SUR UNE PLAQUE VERTICALE

Résumé—Une expression simple pour le nombre de Nusselt (ou de Sherwood) moyen est obtenue à l'aide du modèle de Churchill et Usagi pour tout nombre de Rayleigh et de Prandtl (ou de Schmidt). Au cours des développements il est fait usage de valeurs expérimentales du nombre de Rayleigh tendant vers zéro ou vers l'infini et de solutions théoriques obtenues en théorie de la couche limite laminaire. L'expression est applicable au transfert thermique à flux constant aussi bien qu'à température constante ainsi qu'au transfert de masse et au transfert simultané de chaleur et de masse. La loi de corrélation fournit une base de calcul des taux de transfert pour des fluides non newtoniens et pour des plaques inclinées. Des expressions tout aussi simples sont développées pour des domaines limités correspondant à des conditions particulières. Les expressions d'application générale et d'application restreinte sont comparées aux données expérimentales représentatives. La structure de l'équation de corrélation fait apparaitre la raison pour laquelle les lois habituelles de type puissance ne peuvent s'appliquer sur un domaine étendu de nombres de Rayleigh et de Prandtl.

KORRELATIONEN FÜR LAMINARE UND TURBULENTE FREIE KONVEKTION AN EINER SENKRECHTEN PLATTE

Zusammenfassung—Nach einem Modell von Churchill und Usagi wurde eine einfache Beziehung für mittlere *Nu*-Zahlen (oder *Sh*) für alle *Ra* und *Pr* (oder *Sc*) entwickelt. Es sind dazu experimentelle Werte für *Ra* die gegen Null und unendlich gehen herangezogen und theoretische Lösungen, wie sie aus der Grenzschichttheorie erhalten werden. Die Beziehung ist anwendbar für gleichförmige Heizung, einheitliche Wandtemperatur, für Stoffübergang und gleichzeitigen Wärme- und Stoffübergang. Die Korrelation vermittelt eine Grundlage zur Bestimmung des Übergangs bei nichtnewtonischen Flüssigkeiten und für geneigte Platten. Für bestimmte Anwendungsbereiche werden einfachere Beziehungen angegeben. Die allgemeine Gleichung und die spezielle Beziehung werden vergliechen mit repräsentativen experimentellen Daten. Die Struktur der Korrelationsbeziehung gibt Aufschluß über das Versagen der allgemeinen Exponential-Gleichungen für einen ausgedehnten Bereich von *Ra* und *Pr*.

КОРРЕЛЯЦИОННЫЕ УРАВНЕНИЯ ДЛЯ ОПИСАНИЯ ЛАМИНАРНОЙ И ТУРБУЛЕНТНОЙ СВОБОДНОЙ КОНВЕКЦИИ ОКОЛО ВЕРТИКАЛЬНОЙ ПЛАСТИНЫ

Аннотация — С помощью модели Черчилля и Узаги получено простое выражение для осредненного по пространству значения числа *Nu* (или *Sh*) при любых значениях чисел *Ra* и *Pr* (или *Sc*). При выводе использовались экспериментальные данные для числа *Ra*, стремящегося к нулю и бесконечности, и аналитические решения, полученные на основе теории ламинарного пограничного слоя. Выражение применимо к случаям постоянного теплового потока, постоянной температуры стенки, а также для описания процессов массообмена и одновренемного тепло- и массообмена. Корреляция дает возможность рассчитать скорости переноса в неньютоноских жидкостях и в случае наклонных пластин. Аналогичные, но более простые выражения получены для ограниченных диапазонов условий. Общее и частные выражения сравниваются на достоверных экспериментальных данных. Структура корреляционного уравнения позволяет объяснить тот факт, почему обычные уравнения типа степенных зависимостей не могут успешно применяться при больших диапазонах значений *Ra* и *Pr*.